6.4

Exploration 1

$y = y_0 e^{\underline{k}t}$

$y = y_0 b^{\underline{n}t}$

$3 \cdot 2^{0.7x}$

$3e^{\underline{0.7\ln(2)}x}$

$3(2)^x$

$3e^{\underline{\ln(2)}x}$

$3 \cdot 2^x$

$\frac{3}{3} \cdot \frac{1}{2} + \frac{2}{6}$

$3/6$

This observation is *Newton's Law of Cooling*, although it applies to warming as well, and there is an equation for it.

If T is the temperature of the object at time t, and T_s is the surrounding temperature, then

$$\frac{dT}{dt} = -k(T - T_s). \tag{1}$$

$y = T - T_s$

$\frac{dy}{dt} = -ky$

$\int \frac{dy}{y} = \int -k\,dt$

$\ln y = -kt + C$

$y = Ce^{-kt}$

$y = y_0 e^{-kt}$

Since $dT = d(T - T_s)$, Equation 1 can be written as

$$\frac{d}{dt}(T - T_s) = -k(T - T_s).$$

Its solution, by the law of exponential change, is

$$T - T_s = (T_0 - T_s)e^{-kt},$$

where T_0 is the temperature at time $t = 0$. This equation also bears the name **Newton's Law of Cooling.**

EXAMPLE 6 Using Newton's Law of Cooling

A hard-boiled egg at 98°C is put in a pan under running 18°C water to cool. After 5 minutes, the egg's temperature is found to be 38°C. How much longer will it take the egg to reach 20°C?

SOLUTION

…ing Newton's Law of Cooling with $T_s = 18$ and $T_0 = 98$, we have

$$T - 18 = (98 - 18)e^{-kt} \quad \text{or} \quad T = 18 + 80e^{-kt}.$$

$$T - T_s = (T_0 - T_s)e^{-kt},$$

where T_0 is the temperature at time $t = 0$. This equation also bears the name **Newton's Law of Cooling.**

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SOLUTION

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$$T - 18 = (98 - 18)e^{-kt} \quad \text{or} \quad T = 18 + 80e^{-kt}.$$

To find k we use the information that $T = 38$ when $t = 5$.

$$38 = 18 + 80e^{-5k}$$

$$e^{-5k} = \frac{1}{4}$$

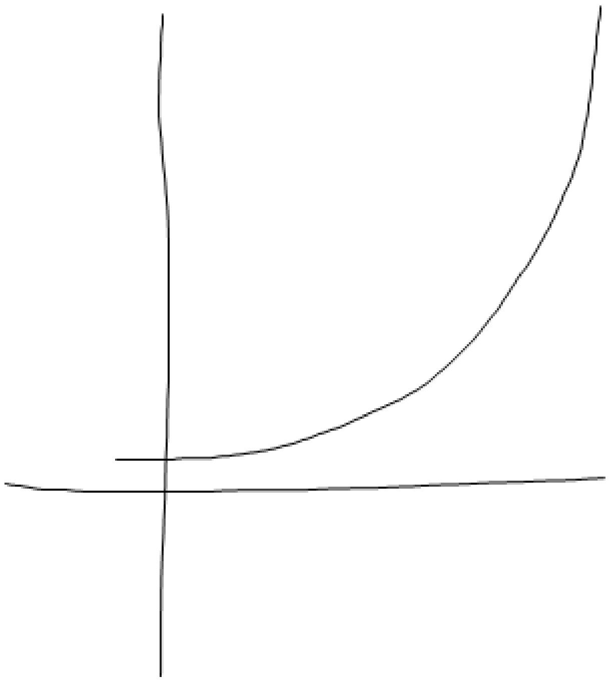
$$-5k = \ln \frac{1}{4} = -\ln 4$$

$$k = \frac{1}{5} \ln 4$$

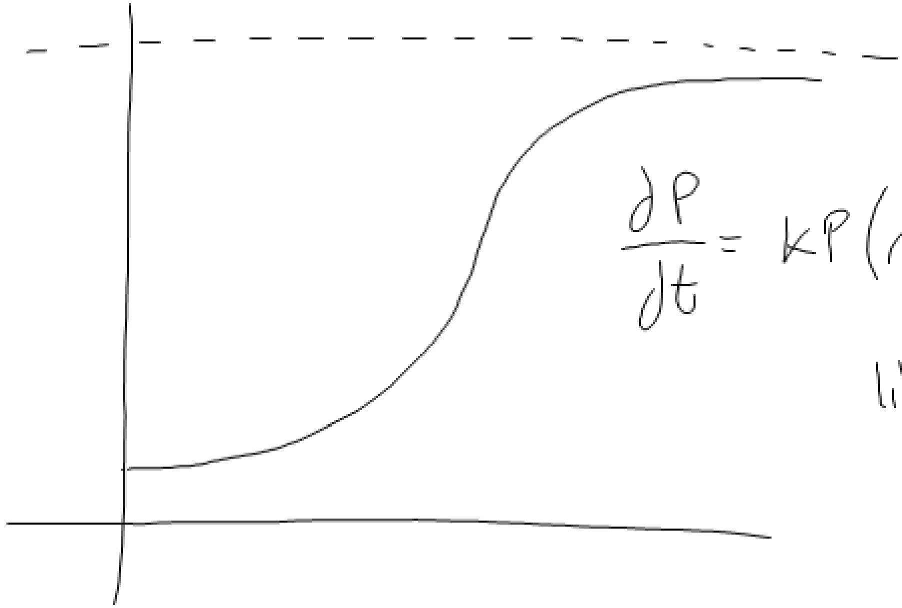
The egg's temperature at time t is $T = 18 + 80e^{-(0.2 \ln 4)t}$.

[Graph]ically We can now use a grapher to find the time when the egg's tem-

Expl. 2
6.5



Logistic curve



$$\frac{dP}{dt} = kP(M-P)$$

limiting factor

$\boxed{6.5}$

#19–22, 23, 26, 31, 32, 37

#21 + 22
long divide
first

No need
to "show"

ex. 6